146

$o(G)=4$

1. Let G be a group and $o(G)=4$. Then G is

abelian

2. If the ring Z of integers is a Euclidean ring then what is d-function d(a) for a in Z?

$|a|$

3. Let Z_n be the set of integers mod n under the addition and multiplication mod n . Then Z_n is a field if n is a

prime number

4. Cauchy -- Riemann equations in polar coordinates are

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \text{ and } \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

5. If $A = \begin{pmatrix} 2 & -1 & 4 \\ 4 & 2 & -3 \\ 1 & 1 & 2 \end{pmatrix}$ then adj A=

$$\begin{pmatrix} 7 & 6 & -5 \\ -11 & 0 & 22 \\ 2 & -3 & 8 \end{pmatrix}$$

6. If G is a group of order 63, then the number of 7- Sylow sub groups in G is

1

7. Let V be a vector space over a field F. If $u, v \in V$, then $|(u,v)| \leq \|u\| \|v\|$. Name this inequality.

Schwarz inequality

8. Let Z_6 be the ring of integers mod 6 under the addition and multiplication mod 6. Then the zero divisors with respect to multiplication mod 6 are

$\bar{2}, \bar{3}$

9. The function $f(z) = e^{\bar{iz}}$ is analytic

nowhere

10. If 0 is a characteristic root of a matrix then the matrix is

singular

11. The number of non – abelian groups of order 8 is

2

12. Two Vectors u,v in a vector space V over a field F, are said to be orthogonal if

$(u, v) = 0$

13. The characteristic of the ring of integers is

0

14. If $F(\alpha) = \oint \frac{4z^2 - 5z + 2}{z - \alpha}$ over C, where C is the ellipse $9x^2 + 16y^2 = 144$ then F(2) ____

16πi

15. If 0, 3, 15 are the characteristic roots of the matrix A = $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$ then the

characteristic roots of the matrix A – 5I are

-5, -2, 10

16. If S = {1,2,3,4,5,6,7,8,9} and α = (1 5 3)(1 2) , β = (1 6 7 9) are permutations in

S_3 then $\alpha\beta\alpha^{-1}$ =

(2 6 7 9)

17. If R^n is a finite dimensional vector space over R and $R^n \cong L(S)$ then S =

S = {(1,0,.....0), (0,1,0,0..........0), (0,0,1...........0),.......(0,0,..........0,1)}

18. The generators of the ring of integers are

1, -1

19. The Taylor series expansion of $f(z) = \frac{1}{1+z^2}$ at z = 0 is

$\sum_0^\infty (-1)^n z^{2n}, \ |z| < 1$

20. The characteristic roots of the matrix $A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$ are

2, 2, 4

21. Let A_n be the set of all even permutations in S_n . If $n > 1$, the order of A_n is

$\frac{1}{2} n!$

22. If V is an inner product space and $v \in V$ then the length of v is defined by

$\|v\| = \sqrt{(v, v)}$

23. Let J be the ring of integers and J_n be the ring of integers modulo n. If $\phi : J \to J_n$ is defined by $\phi(a)$ = reminder of a on division by n ,then find the kernel of ϕ

All multiples of n

24. All zeros of f(z) = zsinz and their orders are

z = 0 is a zero of second order and z = $\pm k\pi, k = 1, 2$........ are zeros of first order

25. The characteristic roots of the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 2 \end{pmatrix}$ and its inverse A^{-1}, are respectively,

-2, 5; -1/2, 1/5

26. The even permutations of S_3 other than identity are

(1 2 3) , (1 3 2)

27. In an inner product space V over a field F, for *u, v, w* in V and α, β in $F, (u, \alpha v + \beta w) =$

$\bar{\alpha}(u,v) + \bar{\beta}(u,w)$

28. The maximal ideals in the ring of integers are

Ideals generated by prime numbers

29. Determine and classify all singularities of the function $f(z) = \frac{1}{\sin\left(\frac{\pi}{z}\right)}$

$z = \pm\frac{1}{n}, n = 1, 2 \ldots\ldots$ are simple poles; $z = 0$ is a non- isolated singular point.

30. The rank of the matrix $A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 2 & 1 \\ 1 & -5 & 8 & -7 \end{pmatrix}$ is

2

31. The number of non- trivial normal subgroups in S_n for $n > 4$ is

1

32. If v is a non-zero vector in an inner product space V over a field F then

$\frac{v}{\|v\|}$ **is a unit vector**

33. Let R be a commutative ring with unit element and M an ideal of R. Then the condition for M to be a maximal ideal of R is

R/M is a field

34. In the annulus $0 < |z| < 1$ and at $z = 0$, the Laurent series expansion of the function $f(z) = \frac{1}{z(1-z)}$ is

$\frac{1}{z} + \sum_{n=0}^{\infty} z^n$

35. A general solution of a linear differential equation with constant coefficients is

a sum of particular solution and complimentary function

36. Let H and K be finite subgroups of a group G. Then the order of HK is

$\left|\frac{H||K}{H\cap K}\right|$

37. Let C be the vector space of complex numbers over R. Let $v_1 = 1, v_2 = 0$. Then

v_1, v_2 **are linearly independent over reals**

38. Let p be a prime element in a Euclidean ring R and p / ab where a, b∈R. Then

p divides at least one of a or b

39. The singularities of the function $f(z) = \frac{z - \sin z}{z^2}$ are

z = 0 is removable singularity

40. Let S = {(1, 1, 1), (0, 1, 1), (0, 0, 1)} = $\{v_1, v_2, v_3\}$ be a basis of R^3 over R. Then the vector (4, -6, 5) in R^3 in terms of the basis is

$4v_1 - 10v_2 + 11v_3$

41. Every subgroup of an abelian group is a

Normal sub group

42. Let V be an inner product space over a field F and W a subspace of V. Then

$V = W + W^{\perp}$

43. If F[x] is a polynomial ring over F and f(x), g(x) are two non-zero elements of F[x] then deg (f(x), g(x)) =

deg f(x) + deg g(x)

44. The Taylor's series expansion of $f(z) = \tan^{-1} z$ at $z_0 = 0$ is

$z - \frac{z^3}{3} + \frac{z^5}{5} - \ldots \ldots \quad |z| < 1$

45. Let G be the group of real numbers under addition. If $\phi : G \to G$ is defined by $\phi(x) = x+1$ for all $x \in G$ then $\ker \phi =$

{-1}

46. Let V be a non-zero finitely generated vector space over a field F. Then

V has a finite basis

47. Let R be a ring and every x in R satisfies $x^2 = x$. Then R is a

commutative ring

48. The residue at poles of the function $f(z)$ is $\frac{\sin z}{z^2}$ is

1

49. Let G be a group of order $11^2 \cdot 13^2$. Then the number of 11- Sylow sub groups and 13-Sylow sub groups of G respectively are.

1 , 1

50. Let V be a finite dimensional vector space over a filed F and W be a 4-dimensional subspace of V. If the dimension of V/W = 15, then dim V =

19

51. Let R be a ring with unit element. Then each maximal ideal in R is a

prime ideal

52. The residue at the poles of the function $f(z) = \frac{1-e^{2z}}{z^4}$ is

$-4/3$

53. If G is a group of order p^2, where p is a prime number, then G is

abelian

54. Let R[x] be the real vector space of polynomials over the field R and defined on [0, 1]. Then

R[x] is an infinite dimensional vector space

55. Which of the following is a field?

Z/3Z

56. Let $f(z)$ be continuous on a domain G and suppose that $\int f(z)\,dz = 0$ for every closed rectifiable curve L contained in G. Then $f(z)$ is analytic on G. Name this theorem,

Morera's theorem

57. Every Group is isomorphic to a sub group of A(S) for some appropriate S. Name this theorem

Cayley's theorem

58. The dimension of the vector space C over R is

2

59. If the polynomial ring F[x] is a Euclidean ring then the d-function d(f(x)) for f(x) F[x] is

deg f(x)

60. Evaluate $\oint \frac{e^z}{z(1-z)^3}$ over C, where C is $|z| = \frac{1}{2}$

2Πi

61. Let $G = \{1, -1\}$ be a multiplicative group and $\mathbb{Z}$ the group of integers. Define $\phi : \mathbb{Z} \to G$

by

$\phi(x) = 1$ if n is even
$\phi(x) = -1$ if n is odd

Then

Φ is a homomorphism

62. Let (1, 3, 5) and (2, 1, 0) be two vectors in the vector space R^3 over R. If (7, m, 5) is a linear combination of the above two vectors then m =

6

63. The maximal ideals in the polynomial ring F[x] are

ideals generated by irreducible polynomials over F

64. The $f(z) = z^2 \bar{z}$ is

no where differentiable.

65. Which of the following polynomials are reducible over C, the field of complex numbers?

$x^2 + 1$

66. Let U and W be finite dimensional subspaces of a vector space V over a field F. If dim U = 6, dim W = 8 and dim (U+W) = 10, then dim $(U \cap W)$ =

4

67. What is a Gaussian integer ?

a complex number a + ib such that a, b are integers

68. Suppose (a, b) = 1. If a/c, b/c then which of the following is true?

ab/c

69. Which of the following is true?

C is a vector space over R

70. What is the dimension of the vector space R ?

Infinite

71. The real valued function that is uniformly continuous on (0, 1) is

$\sin \frac{1}{x}$

72. The solution of ODE $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is

$$e^y = \frac{x^3}{3} + e^x + c$$

73. A solution of the congruence $5x \equiv 3 \pmod{24}$ is given by

$$x \equiv 15 \pmod{24}$$

74. A finite set has

no limit point

75. The sequence $\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots\cdots + \frac{1}{2n}\right)$ is

bounded and convergent

76. The function f defined by $f(x) = \frac{1}{2^n}$, where $\frac{1}{2^{n+1}} < x \leq \frac{1}{2^n}$, $n = 0, 1, 2 \ldots$

$f(x) = 0$

is integrable on

$[0, 1]$

77. The partial differential equation obtained by eliminating a and b from $z = (x-a)^2 + (y-b)^2$ is

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4z$$

78. If p is a prime and $x! \equiv -1 \pmod{p}$ then $x =$

$p-1$

79. If $2^n + 1$ is a prime then n is

a power of 2

80. Bolzano-Weierstrass theorem(for sequences) states

every bounded sequence has a limit point

81. The sum of the divisors of 14553 is

27360

82. $\sum_{n=1}^{\infty} n^3$ is

divergent

83. The p – discriminant relation of $y = px + \frac{1}{p}$ is

$y^2 = 4x$

84. Let X be a metric space and $E \subset X$. Then $E = \bar{E}$ if and only if
E is closed

85. The set $\left\{\frac{1}{n}, n \in N\right\}$ is

neither closed nor open

86. The uncountable set is

The set of real numbers

87. Ordinary differential equation of order n contains

n arbitrary constants

88. The solution of partial differential equation $z = xp + yq$ is

$f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$

89. If $G_n = \left(-\frac{1}{n}, \frac{1}{n}\right)$, $n = 1, 2, \ldots$ is an open subset of R then $\bigcap_{n=1}^{\infty} G_n =$

0

90. The highest power of 2 in 14! is

11

91. Solution of $\log\left(\frac{dy}{dx}\right) = ax + by$ is

$$-\frac{1}{b}e^{-by} = \frac{1}{a}e^{ax} + c$$

92. Eliminating the arbitrary functions *f* and *g* from the equation $y = f(x-at) + g(x+at)$, the partial differential equation obtained is

$$a^2\frac{\partial^2 y}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$$

93. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n =$

e

94. If the set of complex numbers C is a metric space then for $z_1, z_2 \in C$ the metric d is defined on C by

$|z_1 - z_2|$

95. A homogenous differential equation of first order and first degree is of the form

$$\frac{dy}{dx} = f(x, y)$$

96. Eliminating a and b from the equation $z = (x+a)(y+b)$ the partial differential

equation obtained is

$z = pq$

97. If the series $\sum \frac{1}{n^p}$ converges then

$p > 1$

98. The set $A = \frac{1+i}{n}, n \in N$ in the metric space (C, d), where C is the set of complex numbers and d is the usual metric on C is

neither open nor closed

99. The number of divisors of 67375 is

24

100. The series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is

convergent

101. The ordinary differential equation $(y - x^3)dx + (x + y^3)dy = 0$ is

an exact differential equation

102. The function f defined on R by $f(x) = \begin{cases} x, & x \text{ is irrational} \\ -x, & x \text{ is rational} \end{cases}$ is

continuous at x = 0

103. The metric space X = (0, 1] on the real line with the usual metric becomes a complete metric space if

0 is adjoined to X

104. The integrating factor of $(3x^2 - y^2)dy - 2xy\,dx = 0$ is

y^{-4}

105. The function $f(x) = \frac{1}{x}$ is not uniformly continuous on

(0, 1]

106. The partial differential equation $(2x + 3y)p + 4xq - 8pq = x + y$ is

non-linear

107. The function $f(x) = x^2$ is uniformly continuous on

[-1, 1]

108. A complete metric space is a metric space in which every

Cauchy sequence converges

109. The general solution of Clairaut's equation $y = px + f(p)$ is

$$y = cx + f(c)$$

110. The left and right derivatives of

$$f(x) = \{ x \tan^{-1}\frac{1}{x},\ x \neq 0;$$
$$= 0 \text{ if } x = 0$$

are respectively

$$-\frac{\pi}{2}, \frac{\pi}{2}$$

111. An ordinary linear homogeneous differential equation of n^{th} order has

no singular solution

112. The gcd of 2210, 493 is

17

113. A solution of an ordinary differential equation , which contains no arbitrary constants is called

a particular solution

114. Cauchy's method of characteristics is used to solve a partial differential equation which is

non-linear

115. The value of $\int_0^1 \frac{1}{\sqrt{x}}\, dx$ is

2

116. A general solution of a linear differential equation with constant coefficients is

a sum of particular solution and complimentary function

117. The solution of $p \tan x + q \tan y = \tan z$ is

$$\frac{\sin x}{\sin y} = \phi\left(\frac{\sin y}{\sin z}\right)$$

118. Let X be an infinite set with the discrete metric. Then (X, d) is

not compact

119. If (a, b) = 1 then (a + b, a -b) =

either 1or 2

120. There exists a real continuous function on the real line which is

nowhere differentiable

121. In order that $f_n(x) = \dfrac{1}{nx+1}$, $0 < x < 1$, $n = 1, 2 \ldots$ converges uniformly, the condition required is

f_n **must be defined on [0, 1]**

122. Every closed and bounded subset of the real line is compact . Name this theorem

Heine – Borel theorem

123. ~~If a and then (a,b) =~~ If $a = p_1^2 p_2^2 p_3^5 p_4 p_5^3$ and $b = p_1^3 p_2 p_3^4 p_4^2 p_5$ then $(a,b) =$

$p_1^2 p_2 p_3^4 p_4 p_5$

124. The general solution of $p = \log(px - y)$ is

$y = cx - e^c$

125. The solution of $z = p + q$ is

$f(x - y, y - \log z) = 0$

126. The limit f of a sequence of continuous functions $\{f_n\}$ is continuous if

$f_n \to f$ **uniformly**

127. Let f be a monotonic function on (a, b). Then the set of points of (a, b) at which f is discontinuous is

at most countable

128. Applying Charpit's method, a complete integral of $q = 3p^2$ is

$z = ax + 3a^2y + b$

129. Let X be a topological space and Y a non – empty subset of X. Then the relative topology on Y is the class of all

intersections with Y of open sets in X

130. If ϕ is Euler totient function then integers n such that $\phi(n) = \frac{n}{2}$ are

$2^n, n \geq 1$

131. A continuous function from a compact metric space into a metric space is

uniformly continuous

132. The c – discriminant relation of $8ap^3 = 27y$ is

$ay^2 = 0$

133. Let $\sum_{n=1}^{\infty} f_n(x)$ be an infinite series converging uniformly to a function f and f_n's are Riemann integrable. Then f is

Riemann integrable

134. How many solutions does the congruence $x^2 \equiv 1 \pmod 8$ has?

4

135. Let (R, d) be a metric space where R is the set of real numbers and d is the usual metric on R. If f is a function from R to R defined by f(x) = sin x, then its domain and codomain in (R, d) respectively are

$(0, 2\pi)$, $[-1, 1]$

136. If $D = \{(x, y): x \neq 0, y = \sin\frac{1}{x}\}$ is a subset of R^2, then D is

disconnected

137. Let m = 7 and b = 12. If $a \equiv b \pmod 7$ then $ac \equiv bc \pmod{mc}$, where c =

4

138. Components of a metric space are

closed

139. If X is a discrete space then the topology on X is

all subset of X

140. The integrating factor of $x\,dy - y\,dx = 0$ is

$\frac{1}{xy}$

141. If f is a continuous function on [a, b] and $f'(x) \geq 0$ in (a,b) then

***f* is increasing in [a, b]**

142. How many solutions does the congruence $x^2 \equiv 2 \pmod{134}$ has?

No Solution

143. If $p = 29$ then find x such that $x^2 \equiv -1 \mod p$.

17

144. Singular solution of an ordinary differential equation

does not contain an arbitrary constant

145. The open base on R^2 is a class of

open strips, open rectangles, ϕ, R^2

146. How many solutions does the congruence $2x \equiv 3 \pmod 4$ has

No solution

147. Let R be a Boolean ring. Then every prime ideal ~~is a~~ $P \neq R$ is a

maximal ideal

148. The function ~~(x) = |x| is~~ $f(x) = |x|$ is

differentiable at x = 0

149. An open sub base for the real line R consists of all

open intervals $(a, +\infty)$, $(-\infty, b)$, $a, b \in R$

150. The curves passing through (0,1) and satisfying $\sin\left(\frac{dy}{dx}\right) = c$ are

$$\sin\left(\frac{y-1}{x}\right) = c$$