Notations:
1. Options shown in green color and with ✔ icon are correct.
2. Options shown in red color and with ✗ icon are incorrect.

The solution of the ordinary differential equation \( \frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2 \) is:

Options:

1. ✗ \( \frac{1}{x \log x} = \frac{1}{2x^2} + c \)

2. ✗ \( \frac{1}{y \log x} = \frac{1}{2x^2} + c \)
3. \[ \frac{1}{y \log y} = \frac{1}{2x^2} + c \]

4. \[ \frac{1}{x \log y} = \frac{1}{2x^2} + c \]

**Question Number : 2  Question Id : 2310982402  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical**

Correct Marks : 2 Wrong Marks : 0.66

If the differential equation \( x \left( \frac{\partial y}{\partial x} \right)^2 - (x-3)^2 = 0 \) has p-discriminant relation as \( x(x-3)^2 = 0 \) and e-discriminant relation as \( x(x-9)^2 = 0 \), then the singular solution is:

Options:

1. \( x - 3 = 0 \)

2. \( x - 9 = 0 \)

3. \( x = 0 \)

4. \( x(x-3)(x-9) = 0 \)

**Question Number : 3  Question Id : 2310982403  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical**

Correct Marks : 2 Wrong Marks : 0.66

For the ordinary differential equation \( x^2(x-1)^2 y' - (x^2 - 1) \cos xy' - 2 \cos^2 xy = 0 \), which of the following is true?

Options:

1. 0 is regular singular point and 1 is irregular singular point

2. 0 is irregular singular point and 1 is regular singular point

3. 0 and 1 are irregular singular point
4. 0 and 1 are regular singular point

The orthogonal trajectories of the system of curves \( \left( \frac{dy}{dx} \right)^2 = \frac{a}{x} \) are:

Options:

1. \( 9a(y+c)^2 = 4x^3 \)

2. \( 9a(y-c)^2 = 4x^3 \)

3. \( 4a(y+c)^2 = 9x^3 \)

4. \( 4a(y-c)^2 = 9x^3 \)

If \( P_n(x) \) is the solution of Legendre’s second order differential equation, then \( \int_{-1}^{1} x P_n(x) P_1(x) dx \) is equal to:

Options:

1. \( \frac{6}{35} \)

2. \( \frac{4}{15} \)

3. \( \frac{6}{15} \)
Question Number : 6  Question Id : 2310982406  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let \( P_n(x) \) be the solution of Legendre's second order differential equation. If the polynomial \( 2 - 3x + 4x^2 \) is expressed in terms of \( P_n(x) \), then the coefficient of \( P_n(x) \) is:

Options :
1. \(-3\)
2. \(2\)
3. \(\frac{10}{3}\)
4. \(-\frac{10}{3}\)

Question Number : 7  Question Id : 2310982407  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
If \( J_\alpha(x) \) be the solution of Bessel equation of second order differential equation, then \( \int x^4 J_\alpha(x) \, dx \) is equal to:

Options :
1. \( x^4 J_2 - 2x^3 J_3 + c \)
2. \( x^4 J_2 + 2x^3 J_3 + c \)
3. \(-x^4 J_2 + 2x^3 J_3 + c \)
4. \(-x^4 J_2 - 2x^3 J_3 + c \)
Question Number : 8  Question Id : 2310982408  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
If the rectangle \( R \) contains the interval of existence of the solution of the initial value problem \( y' = y^2 + \sin^2 y, y(0) = 0 \), where \( R = \left\{ (x,y) : 0 \leq x \leq k, |y| < M, k > \frac{1}{2}, M > 0 \right\} \), then the interval is:  
Options :  
1. \( 0 < x < \frac{1}{2} \) 
2. \( 0 \leq x \leq \frac{1}{2} \) 
3. \( 0 \leq x \leq 1 \) 
4. \( 0 \leq x < \frac{1}{2} \) 

Question Number : 9  Question Id : 2310982409  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
The integral surface of \( -x^2 p + y^2 q = z^2 \) which passes through \( 2xy = x + y, 4z + 2 = 0 \) is: 
Options :  
1. \( yz + zx - 6xy = 2xyz \) 
2. \( yz + zx + 2xy = 6xyz \) 
3. \( yz + zx - 2xy = 6xyz \) 
4. \( yz + zx + 6xy = 2xyz \) 

Question Number : 10  Question Id : 2310982410  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66
If the partial differential equation \( \frac{\partial z}{\partial x} = x - \frac{y}{(ax^2 + y^2)} \) and \( \frac{\partial z}{\partial y} = y + \frac{x}{(x^2 + \beta^2)} \) are compatible, then \( \alpha^2 + \beta^2 \) is equal to:

1. 2
2. 4
3. 0
4. 5

The complete integral of \( 2pxy + pqy - 2yz \) is:

Options:
1. \((z + ax)(a + y)^2z = be^{2y}\)
2. \((z - ax)(a + y)^2z = be^{2y}\)
3. \((z - ax)(a + y)^2z = be^{-2y}\)
4. \((z + ax)(a + y)^2z = be^{-2y}\)

If \( z = e^{ax} \phi_y(y-x) + e^{by} \phi_x(y-x) + e^{2xy} \) is the general solution of the partial differential equation \( \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y} = e^{2xy} \), then \( a + b + c \) is equal to:

Options:
1. \(-\frac{5}{6}\)
Question Number : 13  Question Id : 2310982413  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

If $\frac{\partial^2 z}{\partial u^2} + b \frac{\partial^2 z}{\partial u \partial v} + c \frac{\partial^2 z}{\partial v^2} = 0$ is canonical form of the partial differential equation $3 \frac{\partial^2 z}{\partial x^2} + 10 \frac{\partial^2 z}{\partial x \partial y} + 3 \frac{\partial^2 z}{\partial y^2} = 0$, then $a+b+c$ is equal to:

Options :
1. ✗ 1
2. ✗ 0
3. ✗ -1
4. ✗ 2

Question Number : 14  Question Id : 2310982414  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

The singular solution of the partial differential equation $z = 2px - 3qy + \log(pq)$ is:

Options :
1. ✗ $z = 2 - \log(\delta xy)$
2. ✗ $z = -2 - \log(\delta xy)$
3. \[ z = 2 \log(\delta xy) \]

4. \[ z = -2 \log(\delta xy) \]

Question Number : 15  Question Id : 2310982415  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
The general solution of the partial differential equation \( 3\frac{\partial^2 z}{\partial x^2} - 10\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = (x + y) \) is:

Options :
1. \[ z = \phi_1(y + 3x) + \phi_2(3y + x) - \frac{1}{24}(x + y)^3 \]  
2. \[ z = \phi_1(y + 3x) - \phi_2(3y + x) - \frac{1}{24}(x + y)^3 \]  
3. \[ z = \phi_1(y + 3x) + \phi_2(3y + x) + \frac{1}{24}(x + y)^3 \]  
4. \[ z = \phi_1(y + 3x) - \phi_2(3y + x) + \frac{1}{24}(x + y)^3 \]

Question Number : 16  Question Id : 2310982416  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
The surface passing through two lines \( z = 0, x = 0 \) and \( z = 0, y = 0 \) and satisfying the partial differential equation \( \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0 \) is:

Options :
1. \[ 2x = y(z - x) \]  
2. \[ 2x = z(y - x) \]
The characteristic curve of the partial differential equation \(2y \frac{\partial u}{\partial x} + (2x + y^2) \frac{\partial u}{\partial y} = 0\) passing through \((0, 1)\) is:

Options:

1. \(y^2 = -2x - 2 + 3e^x\)

2. \(y^2 = 2x + 2 - 3e^x\)

3. \(y^2 = 2x - 2 + 3e^x\)

4. \(y^2 = 2x + 2 + 3e^x\)

If \(J_n(x)\) is the solution of Bessel differential equation, then \(\int_0^\infty e^{-2x}J_0(3x)dx\) is equal to:

Options:

1. \(\frac{1}{\sqrt{5}}\)

2. \(\frac{1}{\sqrt{13}}\)
3. \( \frac{1}{\sqrt{6}} \)

4. \( \frac{1}{\sqrt{3}} \)

**Question Number : 19  Question Id : 2310982419  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical**

Correct Marks : 2  Wrong Marks : 0.66

Let \( u(x) \) be a continuity differentiable function taking non-negative values for \( x > 0 \) and satisfying \( u'(x) = 4u^{3/2}(x) ; u(0) = 0 \), then the differential equation has:

Options :

1. a unique solution

2. two solutions

3. no solution

4. infinite number of solutions

**Question Number : 20  Question Id : 2310982420  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical**

Correct Marks : 2  Wrong Marks : 0.66

If \( y(x) \) be a continuous solution of initial value problem \( y' + 2y = f(x), y(0) = 0 \), where \( f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases} \), then \( y\left(\frac{3}{2}\right) \) is equal to:

Options :

1. \( \frac{\sinh(1)}{e^3} \)

2. \( \frac{\cosh(1)}{e^3} \)
If \( y(x) = xe^x \) be a solution of \( y'' + ay' + by = 0 \), \( a, b \in \mathbb{R} \), then:

Options:

1. \( a > 0, b < 0 \)

2. \( a < 0, b > 0 \)

3. \( a > 0, b > 0 \)

4. \( a < 0, b < 0 \)

Let \( y_1(x) \) and \( y_2(x) \) form a complete set of solution of differential equation \( y'' - 2xy' + \sin(e^{2x})y = 0 \), \( x \in [0,1] \) with \( y_1(0) = 0, y_1'(0) = 1, y_2(0) = 1, y_2'(0) = 1 \). Then the Wronskian \( W(x) \) of \( y_1(x) \) and \( y_2(x) \) at \( x = 1 \) is:

Options:

1. \( e^2 \)

2. \( e \)

3. \( -e^2 \)
4. $-e$

The solution of the system $x' = Ax, A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is:

Options:
1. $\begin{bmatrix} \sin t - \cos t \\ \sin t + \cos t \end{bmatrix}$
2. $\begin{bmatrix} \cos t - \sin t \\ \sin t + \cos t \end{bmatrix}$
3. $\begin{bmatrix} 2 \sin t - \cos t \\ \sin t \end{bmatrix}$
4. $\begin{bmatrix} 2 \sin t + \cos t \\ \sin t \end{bmatrix}$

The critical point of system $\frac{dx}{dt} = -4x - y, \frac{dy}{dx} = x - 2y$, is an:

Options:
1. asymptotically stable node
2. unstable node
3. asymptotically stable spiral
4. unstable spiral

Question Number: 25  Question Id: 2310982425  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical  Correct Marks: 2  Wrong Marks: 0.66

The solution of Cauchy problem for the first order partial differential equation \( \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = z \) on \( D = \{(x, y, z) | x^2 + y^2 = 1, z > 0\} \) with initial condition \( x^2 + y^2 = 1, z = 1 \) is:

Options:
1. \( z = x^2 + y^2 \)
2. \( z = (x^2 + y^2)^2 \)
3. \( z = (2 - (x^2 + y^2))^{\frac{1}{3}} \)
4. \( z = (x^2 + y^2)^{\frac{1}{3}} \)

Question Number: 26  Question Id: 2310982426  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical  Correct Marks: 2  Wrong Marks: 0.66

A surface passing through \( z = x = 0 \) and \( z = x - y = 0 \) and satisfying the partial differential equation \( \frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 4 \frac{\partial^2 z}{\partial y^2} = 0 \) is:

Options:
1. \( z = \frac{3x}{y + 3x} \)
2. \( z = \frac{3x}{2y + x} \)
3. \( z = \frac{3x}{y + 2x} \)
4. \[ z = \frac{2x}{y + 3x} \]

Question Number : 27  
Question Id : 2310982427  
Question Type : MCQ  
Option Shuffling : Yes  
Negative Marks Display Text : 2/3  
Correct Marks : 2  
Wrong Marks : 0.66

Let \( u = u(x,y) \) be the complete integral of partial differential equation \( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = xy \) passing through the points \((0,0,1)\) and \((0,1,\frac{1}{2})\) in \((x,y,u)\)-space.

Then, the value of \( u(x,y) \) at \((-1,1)\) is:

Options:

1. ✓ 0

2. ✗ 1

3. ✗ 2

4. ✗ 3

Question Number : 28  
Question Id : 2310982428  
Question Type : MCQ  
Option Shuffling : Yes  
Negative Marks Display Text : 2/3  
Correct Marks : 2  
Wrong Marks : 0.66

The solution of one dimensional heat equation \( u_{xx} = \frac{1}{k} u_t, \ 0 \leq x \leq 2\pi, \ t > 0 \) and \( u(0,t) = u(2\pi,t) = 0, \ u(x,0) = \sin^2 x \) is:

Options:

1. ✗ \[ \frac{4}{3} \sin x e^{-kt} - \frac{3}{4} \sin 3x e^{-2kt} \]

2. ✓ \[ \frac{3}{4} \sin x e^{-kt} - \frac{1}{4} \sin 3x e^{-2kt} \]

3. ✗ \[ \frac{3}{4} \sin 3x e^{-kt} - \frac{4}{3} \sin 3x e^{-2kt} \]
4. \[ \frac{-3}{4} \sin x e^{-2\pi} + \frac{1}{4} \sin 3x e^{i\pi} \]

Let \( H_1 \) and \( H_2 \) be finite subgroups of \( G \). If \( O(H_1, H_2) = 2, O(H_1) = 3, O(H_2) = 4 \), then \( O(H_1 \cap H_2) \) is:

Options:

1. \( \times \) 1
2. \( \checkmark \) 6
3. \( \times \) 3
4. \( \times \) 12

The number of elements of order 2 in \( Z_2 \times Z_4 \) is:

Options:

1. \( \times \) 1
2. \( \checkmark \) 3
3. \( \times \) 4
4. \( \times \) 2
Which of following is NOT a normal subgroup?

Options:

1. ✓ The subgroup \( \langle (1,2) \rangle \) of \( S_3 \)

2. ✗ A subgroup \( H \) of \( Q_8 \) Quaternion group

3. ✗ A subgroup \( \mathbb{Z} \) of \( \mathbb{Z} \)

4. ✗ A subgroup \( H \) of group \( G \) of index 2

---

Question Number : 32  Question Id : 2310982432  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2 Wrong Marks : 0.66  

Which of the following is true?

Options:

1. ✗ If \( p : S_3 \to S_3 / A_3 \) is natural homomorphism, then \((1,2) \in \ker p\)

2. ✗ If \( p : S_3 \to S_3 / A_3 \) is natural homomorphism, then \((1,2) \in \text{Im } p\)

3. ✓ If \( p : S_3 \to S_3 / A_3 \) is natural homomorphism, then \((1,2) \notin \text{Im } p\)

4. ✗ \( \mathbb{Z}_n \) and \( \mathbb{Z}/n\mathbb{Z} \) are isomorphic

---

Question Number : 33  Question Id : 2310982433  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2 Wrong Marks : 0.66  

Let \( G \) be a group and \( H \) be a subgroup of \( G \). If \( O(G) = 7 \) and \( a \in H \), then \( a^{135} \) is equal to:

Options:
1. \( e \)

2. \( a \)

3. \( a^2 \)

4. \( a^3 \)

Question Number : 34  Question Id : 2310982434  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Let \( S = \{ z \in \mathbb{C} : |z| = 1 \} \) be a circle group and \( f : \mathbb{R} \rightarrow S \) s.t. \( f(x) = e^{2\pi i x} \), then \( \ker(f) \) is equal to:

Options:
1. \( \{0\} \)

2. \( \mathbb{Z} \times \mathbb{Z} \)

3. \( \mathbb{Z} \)

4. \( \mathbb{N} \)

Question Number : 35  Question Id : 2310982435  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

If \( G \) is a cyclic group of order 12, then the number of \( Aut(G) \) are:

Options:
1. \( 3 \)

2. \( 4 \)
3. 5

4. 6

4. 11

Question Number : 36  Question Id : 2310982436  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Correct Marks : 2  Wrong Marks : 0.66
Options :

1. 5

2. 7

3. 9

4. 11

Question Number : 37  Question Id : 2310982437  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Correct Marks : 2  Wrong Marks : 0.66
Let \( G \) be a group and \( O(\text{Aut}(G)) > 1 \), then the \( O(G) \) satisfies:

Options :

1. \( O(G) = 2 \)

2. \( O(G) > 2 \)

3. \( O(G) = 1 \)

4. \( O(G) < 2 \)

Question Number : 38  Question Id : 2310982438  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Options :
The number of maximal Ideals of \( \mathbb{Z}_6 \) (the ring of integers modulo 6) are:

Options:
1. \( \mathbb{Z} \)
2. \( Z \)
3. \( 2 \)
4. \( 7 \)

The field of quotients of the integral domain \( \mathbb{Z}[i] = \{a + ib; a, b \in \mathbb{Z}\} \) is:

Options:
1. \( \{x + iy; x, y \in \mathbb{Z}\} \)
2. \( \{x + iy; x, y \in \mathbb{Q}\} \)
3. \( \{x + iy; x, y \in \mathbb{R}\} \)
4. \( \{x + iy; x, y \in \mathbb{C}\} \)

Let \( G = \{e, a, a^2, a^3\} \) be a cyclic group of order 4, then the characteristic subgroup of \( G \) is:

Options:
1. \( \{e, a\} \)
Question Number : 41  Question Id : 2310982441  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
The abelian group of order 6 must be cyclic group, if it contains an element of order:

Options :
1. 2
2. 6
3. 3
4. 4

Question Number : 42  Question Id : 2310982442  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
If \( G \) is an infinite cyclic group, then \( G \) has:

Options :
1. three generators
2. infinite generators
3. exactly one generator
4. \( \checkmark \) exactly two generators

The units of \( \mathbb{Z}[\sqrt{-5}] \) are:

Options:
1. \( \checkmark \) \( \pm 1 \)
2. \( \times \) \( \pm 5 \)
3. \( \times \) \( \pm 2 \)
4. \( \times \) \( \pm 3 \)

Let \( G \) be a cyclic group of order 121. Then the order of its group of Automorphism is:

Options:
1. \( \checkmark \) 110
2. 120
3. 121
4. 11

Which of the following is a possible candidate for the characteristics of an integral domain?

Options:
If $F$ is a field, then which of the following is INCORRECT?

Options:
1. $F[x]$ is a Euclidean domain
2. $F[x]$ is a Principal Ideal domain
3. $F[x_1, x_2]$ is a unique factorisation domain
4. $F[x_1, x_2]$ is a Principal Ideal domain

If $f(x) = x^2 + 5x \in \mathbb{Z}_5[x]$, then the number of roots of $f(x)$ in $\mathbb{Z}_5[x]$ is:

Options:
1. 3
2. 4
Which of the following ideals is NOT a Maximal Ideal?

Options:
1. \( \langle x^1 - 1 \rangle \) in \( \mathbb{Q}[x] \)

2. \( \langle s \rangle \) in \( \mathbb{Z} \)

3. \( \langle x^2 + x + 1 \rangle \) in \( \mathbb{R}[x] \)

4. \( M_1 = \{0,3,9\} \) in \( \mathbb{Z}_{12} \)

Let \( F \) be a field such that \( a \in F \). If \( a \) is roots of \( x^7 - x \in F[x] \), then which of the following is true?

Options:
1. \( Z \subseteq F \)

2. \( Q \subseteq F \)

3. \( F \subseteq Q \)
4. \( \sqrt{3} \)

Question Number : 50  Question Id : 2310982450  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66
Let \( G \) be a group of order 7 and let \( f : G \to G \) be defined by \( f(x) = x^4 \). Then \( f \) is:
Options :
1. \( \times \) not one-one
2. \( \checkmark \) not onto
3. \( \times \) not a homomorphism
4. \( \times \) an isomorphism

Question Number : 51  Question Id : 2310982451  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66
The number of elements of order 2 of the symmetric group \( S_5 \) is:
Options :
1. \( \times \) 22
2. \( \times \) 10
3. \( \times \) 20
4. \( \checkmark \) 25

Question Number : 52  Question Id : 2310982452  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66
The order of \( \frac{5}{6} + \mathbb{Z} \) in the quotient group \( \mathbb{Q}/\mathbb{Z} \) of the additive group of rational numbers is:
Options :
1. \[ H = S_4 \]

2. \[ H \text{ is abelian} \]

3. \[ [S_4 : H] = 2 \]

4. \[ H \text{ is cyclic} \]

---

Let \( \mathbb{R} \) be the set of real numbers and let \( f_{(a,b)} : \mathbb{R}^2 \to \mathbb{R}^2 \) be defined by \( f_{(a,b)}(x,y) = (x + a, y + b) \), where \( a, b \in \mathbb{R} \).

Then the set \( G = \{ f_{(a,b)} \mid a, b \in \mathbb{R} \} \) under the composition of mappings is:

1. an abelian group

2. not a group
3. a group but not necessarily abelian

4. a group but not necessarily cyclic

Question Number : 55  Question Id : 2310982455  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
The group \( G \) is abelian if:

Options :
1. \( |G| = p^3 \) for some prime \( p \)

2. every proper subgroup of \( G \) is cyclic

3. every subgroup of \( G \) is normal in \( G \)

4. the function \( f: G \to G \), defined by \( f(x) = x^{-1} \) for all \( x \in G \), is a homomorphism

Question Number : 56  Question Id : 2310982456  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Let \( N \) be a normal subgroup of \( G \). Then which of the following is true?

Options :
1. If \( G \) is non-abelian, then \( G/N \) is also non-abelian

2. If \( G \) is cyclic, then \( G/N \) is abelian

3. If \( G \) is infinite, then \( G/N \) is also infinite

4. If \( G \) is abelian, then \( G/N \) is cyclic

Question Number : 57  Question Id : 2310982457  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
The number of elements of order 15 in alternating group $A_8$ is:

Options:

1. $\frac{8!}{3!} \times 5$

2. $\frac{8!}{5!} \times 2$

3. $\frac{8!}{5!} \times 3$

4. $\frac{8!}{5}$

The order of the centre of a non-abelian group of order 1001 is:

Options:

1. 1

2. 7

3. 13

4. 77

Let $T: M_{2,2} \rightarrow P_2$ be a linear transformation ($M_{2,2}$ and $P_2$ are the real vector spaces of matrices of order 2x2 and polynomials of degree less than or equal to 2, respectively). Then, the bases of the range of $T$ and kernel of $T$ corresponding to the linear transformation

$$T\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_1 + (a_{12} + a_{21})x + a_{22}x^2$$

is:
Which of the following linear transformations is NOT invertible?

Options:

1. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined as $T(x_1, x_2, x_3) = (x_1, x_2, x_3)$

2. $T : P_2 \rightarrow P_2$ is defined as $T(p(x)) = p'(x)$

3. $T : M_{2\times 2} \rightarrow M_{2\times 2}$ is defined as $T \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

4. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is defined as $T \begin{bmatrix} x \\ y \end{bmatrix} = A \begin{bmatrix} x \\ y \end{bmatrix}$, where $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, fixed $\theta \in \mathbb{R}$
If \( T : P_2 \rightarrow P_3 \) is the linear transformation defined by \( T(p(x)) = xp'(x) + \int_0^x p(t) \, dt \) and \( A = \begin{bmatrix} a_{ij} \end{bmatrix} \) is the 4×3 matrix of \( T \) with respect to standard bases, then the value of \( a_{35} + a_{45} \) is:

Options:
1. \( \times \) 0
2. \( \checkmark \) \( \frac{1}{3} \)
3. \( \times \) \( \frac{1}{2} \)
4. \( \times \) 1

Let a linear transformation \( T : \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) be such that \( T(x, y) = (2x + y, y - x, 3x + 4y) \) then \( \text{nullity}(T) \) equals:

Options:
1. \( \times \) 4
2. \( \times \) 3
3. \( \times \) 1
4. \( \checkmark \) 0

Let \( A \) be a square matrix of order 3 with eigenvalues 2, 2 and 3. Then \( A \) will be diagonalizable if \( \text{rank}(A - 2I) \) equals:
Question Number : 64  Question Id : 2310982464  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let \( A \) be a \( 5 \times 5 \) matrix with eigenvalues 1 and \( -1 \) having algebraic multiplicities 2 and 3 and geometric multiplicities 2 and 2, respectively. Then the number of Jordan blocks in Jordan canonical form of \( A \) corresponding to all the eigenvalues is:
Options :
1. ✔ 4
2. ✗ 3
3. ✗ 2
4. ✗ 1

Question Number : 65  Question Id : 2310982465  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let \( A = \begin{pmatrix} 0 & a & 0 \\ 0 & 0 & b \\ 0 & 0 & 0 \end{pmatrix} \), \( a \in \mathbb{R} \), \( b \in \mathbb{R} \). Then, the values of \( a \) and \( b \) for which \( A \) is diagonalizable are:
Options :
1. $\forall a, b \in R$

2. $b = 0$ and $\forall a \in R$

3. $a = 0$ and $\forall b \in R$

4. $a = 0$ and $b = 0$

---

Question Number : 67  Question Id : 2310982467  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Which of the following matrix is NOT diagonalizable?
Options :
1. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
2. \[
\begin{bmatrix}
1 & 0 \\
3 & 2
\end{bmatrix}
\]

3. \[
\begin{bmatrix}
0 & -1 \\
1 & 0
\end{bmatrix}
\]

4. \[
\begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}
\]

Question Number : 68  Question Id : 2310982468  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Which of the following sets of $2 \times 2$ real matrices $M_{2\times 2}$ under the standard component wise addition and scalar multiplication is NOT a vector space? 
Options :

1. $V = \{ A \in M_{2\times 2} \mid \text{trace}(A) = 0 \}$

2. $V = \{ A \in M_{2\times 2} \mid A = A^T \}$

3. $V = \{ A \in M_{2\times 2} \mid \det(A) = 0 \}$

4. $V = \{ A \in M_{2\times 2} \mid A = -A^T \}$

Question Number : 69  Question Id : 2310982469  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
If the dimension of the vector space spanned by the row vectors \{(1,-1,1),(2,1,1),(3,0,\alpha)\} is 2, then the value of \(\alpha\) is:  
Options :

1. 0
Question Number: 70  Question Id: 2310982470  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical  Correct Marks: 2  Wrong Marks: 0.66

Consider two subsets $A_1$ and $A_2$ of vector space $\mathbb{R}^3$ as $A_1 = \{(1,0,0),(1,2,0),(1,2,3)\}$ and $A_2 = \{(1,3,0),(-2,0,3),(0,2,1)\}$ then which of the following forms the basis for $\mathbb{R}^3$?

Options:
1. $A_1$ but not $A_2$
2. $A_2$ but not $A_1$
3. Both $A_1$ and $A_2$
4. Neither $A_1$ nor $A_2$

---

Question Number: 71  Question Id: 2310982471  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical  Correct Marks: 2  Wrong Marks: 0.66

Let $x$ be a real $2 \times 1$ vector satisfying $x^T x = 1$. Define $A = I - 2xx^T$, where $I$ is an identity matrix of order $2 \times 2$. Then which of the following is true?

Options:
1. $\text{trace}(A) = 1$
2. $A$ is singular matrix
3. \( A^2 = I \)

4. \( A^2 = A \)

---

**Question 72**

Question Id: 2310982472  
Question Type: MCQ  
Option Shuffling: Yes  
Negative Marks Display Text: 2/3  
Option Orientation: Vertical  
Correct Marks: 2  
Wrong Marks: 0.66

Consider a linear transformation \( T: \mathbb{R}^2 \to \mathbb{R}^3 \) with \( T(1,1,1) = (1,1,1), T(1,2,3) = (-1,-2,-3) \) and \( T(1,1,2) = (2,2,4) \).

Then \( T(2,3,6) \) equals:

Options:

1. \( (2,1,4) \)

2. \( (2,-1,6) \)

3. \( (-2,1,6) \)

4. \( (-2,-1,4) \)

---

**Question 73**

Question Id: 2310982473  
Question Type: MCQ  
Option Shuffling: Yes  
Negative Marks Display Text: 2/3  
Option Orientation: Vertical  
Correct Marks: 2  
Wrong Marks: 0.66

Let \( A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & 5 \end{pmatrix} \), \( B = I + A + \ldots + A^{10} \) and \( P^{-1}AP = \text{diag}(-2, 5, 1) \). If trace of \( P^{-1}BP = \alpha 5^{11} + \beta 2^{11} + \delta \), then \( 3(\alpha + \beta + \delta) \) equals:

Options:

1. \( 35 \)

2. \( 37 \)

3. \( 39 \)
Consider the polynomial space $P(t)$ with inner product

$$< f, g > = \int_0^1 f(t)g(t)dt$$

Then, which of the following statements is true?

Options:

1. $f_1(t) = 3t - 5$ and $g_1(t) = t^2$ are orthogonal to each other

2. $f_1(t) = 3t - 1$ and $g_1(t) = t$ are orthogonal to each other

3. \[ \left( \frac{3t - 1}{2} \right) \in S^2, \text{ where } S = \left\{ t, t^2 + \frac{1}{6} \right\} \subset P(t) \]

4. If $g(t) = t^2$, then $\| g \|^2$ is $\frac{1}{6}$

Which of the following statements is true for a square matrix $A$ of order 3?

Options:

1. $x(x - 1)(x + 1)$ can be characteristic polynomial of $A$ if $A$ is orthogonal

2. $(x - 2)(x + 1)(x - 1)$ can be characteristic polynomial of $A$ if $A$ is unitary

3. $(x + 1)(x - 1)^2$ can be characteristic polynomial of $A$ if $A$ is idempotent
4. $x(x^2 + 4)$ can be the characteristic polynomial of $A$ if $A$ is skew symmetric

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

\[ T(v_1) = v_1 + v_2, \quad T(v_2) = v_2 + v_3, \quad T(v_3) = v_3 + v_1 \]

Where $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$. Then, which of the following is true?

Options:
1. ✗ $T$ is one-one but not onto
2. ✗ $T$ is onto but not one-one
3. ✓ $T$ is both one-one and onto
4. ✗ $T$ is neither one-one nor onto

Consider a linear operator $T : P_2 \to P_2$ (where $P_2$ is the vector space of all real polynomials of degree at most 2) such that $T(1) = x^2 + x, \ T(x) = x^2 + x + 1, \ T(x^2) = 2x^2 + 3x + 1$.

Then $T^{-1}(x)$ equals:

Options:
1. ✗ $x^2 + x - 1$
2. ✓ $x^2 - x - 1$
3. ✗ $x^2 - x + 1$
4. \( x^2 + x + 1 \)

Let an inner product space on \( \mathbb{R}^2 \) be defined as \( <u, v> = u^T A v \) where
\[
A = \begin{bmatrix} 1 & -1 \\ -1 & k \end{bmatrix}, \quad k \in \mathbb{R}
\]
Then \( k \in: \)

Options:

1. \( (1, \infty) \)

2. \( (0, \infty) \)

3. \( (0,1) \)

4. \( (-1, \infty) \)

Question Number : 79  Question Id : 2310982479  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Which of the following statements is FALSE for a linear transformation \( T : V \rightarrow W \), where \( V \) and \( W \) are finite dimensional vector spaces?

Options:

1. \( \dim V < \dim W \Rightarrow T \text{ is not onto} \)

2. \( \text{nullity}(T) = 1 \Rightarrow T \text{ is not one-one} \)

3. \( \dim V > \dim W \Rightarrow T \text{ is one-one} \)

4. \( T \text{ is invertible} \Rightarrow T \text{ carries linearly independent sets of } V \text{ onto linearly independent sets of } W \)
Question Number: 80  Question Id: 2310982480  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical
Correct Marks: 2  Wrong Marks: 0.66
Let $U$ and $W$ be two subspaces of the vector space $\mathbb{R}^8$ having dimensions 6 and 5, respectively. Then the minimum value of dimension of $U \cap W$ is:

Options:
1. $\times$ 1
2. $\checkmark$ 3
3. $\times$ 5
4. $\times$ 6

Question Number: 81  Question Id: 2310982481  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical
Correct Marks: 2  Wrong Marks: 0.66
Consider the set $W = \{(1,-2,0,0)^T,(-1,3,1,-1)^T,(0,-1,0,-1)^T\}$ of vectors in $\mathbb{R}^4$. A vector $(\alpha, \beta, \gamma, \delta) \in \mathbb{R}^4$ will be orthogonal to $W$ if:

Options:
1. $\times$ $\alpha - \beta = 0$ and $\gamma - \delta = 0$
2. $\times$ $\alpha + \beta = 0$ and $\gamma + \delta = 0$
3. $\checkmark$ $\alpha + \gamma = 0$ and $\beta + \delta = 0$
4. $\times$ $\alpha - \gamma = 0$ and $\beta + \delta = 0$

Question Number: 82  Question Id: 2310982482  Question Type: MCQ  Option Shuffling: Yes  Negative Marks Display Text: 2/3  Option Orientation: Vertical
Correct Marks: 2  Wrong Marks: 0.66
Which of the following statements is NOT true for an orthogonal matrix $P$ and an inner product space $V$?

Options:
1. \( P^{-1} \) is also an orthogonal matrix

2. \((PQ^{-1})^T\) may not be orthogonal, when \(Q\) is orthogonal

3. \( <Pu, Pv> = <u, v>\) for every \(u, v \in V\)

4. \( ||Pu|| = ||u||\) for every \(u \in V\)

Question Number : 83  Question Id : 2310982483  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let \(A\) be a square matrix of order 2 having eigenvalues 1 and 2. Then the determinant of the matrix \(A^2 + A^{-1} + 2I\) equals:
Options :
1. \(24\)
2. \(26\)
3. \(27\)
4. \(28\)

Question Number : 84  Question Id : 2310982484  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let \(A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 4 \\ -2 & -3 & 10 \end{bmatrix}\). Then for any \(b \in \mathbb{R}\), the system \(Ax = b\) will be inconsistent when \(\text{rank}(A|b)\) is:
Options :
1. \(0\)
2. 1

3. 2

4. ✓ 3

If a square matrix $A$ is both unitary and hermitian, then $A$ is always equal to:

Options:

1. an Identity matrix $I$

2. $A^2$

3. ✓ $A^{-1}$

4. 2$A + I$

---

Question Number : 86  Question Id : 2310982486  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Let $m(x)$ be the minimal polynomial and $c(x)$ be the characteristic polynomial of $A = \begin{bmatrix} 1 & 2 & -3 \\ 4 & 5 & 0 \\ 3 & 6 & 2 \end{bmatrix}$ and $f(x) = x m(x) - x^2 c(x) + x$. Then the rank of the matrix $f(A)$ is:

Options:

1. ✓ 3

2. 2
3. \* \* 1

4. \* \* 0

Which of the statement is true for a square matrix $A$?

Options:

1. \* Eigenvectors are orthogonal to each other

2. 

If $E_1$, $E_2$ are eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2$, respectively, then $E_1 + E_2$ will be also an eigenvector

3. 

If $E_1$ is the eigenvector corresponding to $\lambda_1$ then this will be an unique eigenvector corresponding to $\lambda_1$

4. \checkmark

Let $V_1 = \{E_1 : AE_1 = \lambda_1 E_1\} \cup \{0\}$ where $\lambda_1$ an eigenvalue of $A$ then, $V_1$ will form a vector space.

Question Number : 88 Question Id : 2310982488 Question Type : MCQ Option Shuffling : Yes Negative Marks Display Text : 2/3 Option Orientation : Vertical Correct Marks : 2 Wrong Marks : 0.66

Consider the vectors $v_1 = (1, 1, 1, 1), v_2 = (1, 1, 2, 4)$ and $v_3 = (1, 2, -4, -3)$. Let the vectors $\{w_1, w_2, w_3\}$ be defined as:

\[ w_1 = v_1 \]
\[ w_2 = v_2 - \alpha v_1 \]
\[ w_3 = v_3 - \beta w_1 - \gamma w_2 \]

Then, the value of $\alpha + \beta + \gamma$ such that the vectors $w_1, w_2, w_3$ are orthogonal to each other is:

Options:

1. \* \* $-\frac{3}{2}$
The minimal polynomial of \[
\begin{pmatrix}
4 & 0 & -3 \\
4 & -2 & -2 \\
4 & 0 & -4
\end{pmatrix}
\] is:

Options:
1. \((x^2 - 4)\)
2. \((x - 2)\)
3. \((x + 2)\)
4. \((x + 2)(x^2 - 4)\)

Let \(A = \begin{pmatrix} -2 & 1 \\ 0 & -1 \end{pmatrix}\) and \(B = A + A^2 + A^3 + A^4 + A^5\). Then \(B\) equals to:

Options:
Calculate the value of \( \lim_{x \to 1^+} \left( \frac{x-1-t}{x^2-2x+2} \right)^2 \)

Options:

1. \( \frac{1}{4} \)
2. \( \frac{-1}{4} \)
3. 4
4. -4

If the function \( e^x (\cos y + i\sin y) \) is holomorphic, then its derivative would be:
1. $e^{x+iy}$
2. $e^{x-iy}$
3. $e^{(x+iy)^2}$
4. $e^{(x-iy)^2}$

Calculate the radius of the convergence of power series $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$.

Options:
1. 0
2. $\infty$
3. 1
4. -1

Which option is true regarding the region of the convergence of the series $\sum_{n=1}^{\infty} n! x^n$.

Options:
1. The region contains only one point, that is, 0
2. The region contains no point
3. The region contains two points, that is, 0 and 1

4. The region contains infinite points

Question Number : 95  Question Id : 2310982495  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66
Calculate the radius of convergence of $\sum \frac{n!}{n} x^n$.
Options :
1. $e$
2. $-e$
3. 0
4. $\infty$

Question Number : 96  Question Id : 2310982496  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66
Calculate the radius of convergence for $\sum (4 + 3i)^n x^n$.
Options :
1. $\frac{1}{5}$
2. 5
3. $\frac{1}{5}$
4. 0
Find the domain of the convergence of the series \( \sum_{n=1}^{\infty} \left( \frac{i(x-1)}{2+i} \right)^n \).

Options:

1. Convergent for the set of values of \( x \) that lie inside the circle of radius \( \sqrt{5} \) and centre at \( x=-i \).

2. Convergent for the set of values of \( x \) that lie inside the circle of radius \( \sqrt{5} \) and centre at \( x=i \).

3. Convergent for the set of values of \( x \) that lie inside the circle of radius \( 2\sqrt{5} \) and centre at \( x=-i \).

4. Convergent for the set of values of \( x \) that lie inside the circle of radius \( -\sqrt{5} \) and centre at \( x=-i \).

---

If \( T_1(x) = \frac{x+2}{x+3} \) and \( T_2(x) = \frac{x}{x+1} \), then the value of \( T_1^{-1}T_2(x) \) would be:

Options:

1. \( x+1 \)

2. \( x-2 \)

3. \( x-1 \)

4. \( x \)
Calculate the value of the residue of function \( \csc x \).

Options:
1. \( \times \) 0
2. \( \checkmark \) 1
3. \( \times \) -1
4. \( \times \) 2

Question Number : 100  Question Id : 2310982500  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Calculate the residues of \( \frac{x^2}{(x-1)(x-2)(x-3)} \) at 1, 2, 3 and infinity respectively.

Options:
1. \( \checkmark \) \( \frac{1}{2} \), -4, \( \frac{9}{2} \) and -1
2. \( \times \) \( -\frac{1}{2} \), -2, \( \frac{9}{2} \) and -1
3. \( \times \) \( \frac{1}{2} \), -2, 3 and -1
4. \( \times \) \( \frac{1}{2} \), -4, \( \frac{3}{2} \) and -1

Question Number : 101  Question Id : 2310982501  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Calculate the residue of \( \frac{x^2}{(x^2-1)} \) at \( x = \infty \).

Options:
1. \( \times \) 0
2. ✗ 1

3. ✓ -1

4. ✗ -1/2

The behaviour of the power series \( \sum_{n=0}^{\infty} \frac{x^{4n}}{1+4n} \) at \( x = \pm 1 \) and \( \pm i \) is:

Options:

1. ✓ Non-convergent at \( x = \pm 1 \) and \( \pm i \)

2. ✗ Convergent at \( x = \pm 1 \) and \( \pm i \)

3. ✗ Non-convergent at \( x = \pm 1 \) but convergent at \( \pm i \)

4. ✗ Convergent at \( x = \pm 1 \) but non-convergent at \( \pm i \)

Find the region of convergence of the series \( \sum_{n=0}^{\infty} \frac{(x+2)^n}{(1+n)^4n} \).

Options:

1. ✓ The radius of convergence is 4 and centre is -2

2. ✗ The radius of convergence is 2 and centre is -4

3. ✗ The radius of convergence is 4 and centre is -1
The radius of convergence is 2 and centre is 2

Question Number : 104  Question Id : 2310982504  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Calculate the orthogonal trajectory of the curves $x^2 - y^2 + x = c$.

Options :
1. $xy + 2y = c$
2. $xy - y = c$
3. $2xy + y = c$
4. $2xy - y = c$

Question Number : 105  Question Id : 2310982505  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3
Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Find the value of the integral $\int_{0}^{1+i} (x - y + ix^2) \, dz$ along the straight line from $z=0$ to $z=1+i$.

Options :
1. $\frac{i}{3}$
2. $\frac{i + 1}{3}$
3. $\frac{i + 1/3}{3}$
4. $\frac{i - 1}{3}$
Find the Taylor series representing the function \( \frac{z^2 - 1}{(z+2)(z+3)} \) in the region \( |z| < 2 \).

Options:

1. \[ \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2^n+1} + \frac{8}{3^n+1} \right) z^n \]

2. \[ 1 + \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2^n+1} + \frac{8}{3^n+1} \right) z^n \]

3. \[ 1 + \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2^n+1} - \frac{8}{3^n+1} \right) z^n \]

4. \[ \sum_{n=0}^{\infty} (-1)^n \left( \frac{3}{2^n+1} - \frac{8}{3^n+1} \right) z^n \]

---

Find the Laurent’s series representing the function \( \frac{z^2 - 1}{(z+2)(z+3)} \) in the region \( 2 < |z| < 3 \).

Options:

1. \[ \sum_{n=0}^{\infty} (-1)^n \left( \frac{3.2^n}{2^n+1} - \frac{8.2^n}{3^n+1} \right) \]

2. \[ 1 + \sum_{n=0}^{\infty} (-1)^n \left( \frac{2.2^n}{2^n+1} + \frac{9.2^n}{3^n+1} \right) \]

3. \[ 1 + \sum_{n=0}^{\infty} (-1)^n \left( \frac{3.2^n}{2^n+1} - \frac{8.2^n}{3^n+1} \right) z^n \]

4. \[ \sum_{n=0}^{\infty} (-1)^n \left( \frac{2.2^n}{2^n+1} - \frac{9.2^n}{3^n+1} \right) \]
Question Number : 108  Question Id : 2310982508  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
What is the singularity of the function $\frac{1}{\sin z - \cos z}$ at $z = \pi/4$? 

Options : 

1. ✓ Simple pole singularity 
2. × Isolated singularity 
3. × Isolated essential singularity 
4. × No singularity 

Question Number : 109  Question Id : 2310982509  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Use Rouche’s theorem to find the number of roots of the equation $z^8 - 4z^5 + z^2 - 1 = 0$ that lie inside a circle $|z| = 1$. 

Options : 

1. × 2 
2. × 3 
3. × 4 
4. ✓ 5 

Question Number : 110  Question Id : 2310982510  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Find the residues of $\frac{z+1}{z^2(z-3)}$. 

Options : 

1. ✓ -4/9 and 4/9
2. ✗ -1/9 and 1/9
3. ✗ -2/9 and 2/9
4. ✗ -5/9 and 7/9

Question Number : 111  Question Id : 2310982511  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
In a finite plane, the function having poles as the only singularity points in the finite part is known as a/an:
Options :
1. ✗ analytic function
2. ✗ entire function
3. ✗ isolated function
4. ✔ meromorphic function

Question Number : 112  Question Id : 2310982512  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Find the number of zeros in the polynomial \( f(z) = z^{10} - 6z^7 + 3z^3 + 1 \) in \( |z| < 1 \) using Rouche's Theorem.
Options :
1. ✗ 3
2. ✗ 5
3. ✔ 7
If the power series $\sum a_n z^n$ is convergent and $\sum |a_n z^n|$ is non-convergent, then the series $\sum_{n=0}^{\infty} a_n z^n$ is said to be:

1. divergent
2. conditionally convergent
3. convergent
4. oscillatory

Calculate the number of isolated singular points of the function $f(z) = \frac{z+3}{z^2(z^2+2)}$.

Options:
1. 1
2. 2
3. 3
4. 4

Find the value of $\int_{0}^{\infty} \frac{dz}{z^a + a^a}$ where $a > 0$. 
Calculate the value of \( \int_0^\infty \frac{\log(1+z^2)}{1+z^2} \, dz \) using Contour integration.

Options:

1. \( \pi \)

2. \( \pi \log 2 \)

3. \( \pi / \log 2 \)

4. \( \log 2 / \pi \)

Find the value of \( p \) for which the series \( \sum \frac{\sin nx}{n^p} \) is uniformly convergent for all \( x \in \mathbb{R} \).

Options:

1. \( 0 \)
2. ✓ 1
3. ✗ -1
4. ✗ \infty

Question Number : 118  Question Id : 2310982518  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical Correct Marks : 2 Wrong Marks : 0.66
Which of the following options is correct about the function \( f(z) = \sin x \cosh y + i \cos x \sinh y \)? Options:
1. ✓ The function is continuous as well as analytic everywhere.
2. ✗ The function is neither continuous nor analytic everywhere.
3. ✗ The function is not continuous at some points but analytic everywhere.
4. ✗ The function is continuous everywhere but not analytic at the origin.

Question Number : 119  Question Id : 2310982519  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3 Option Orientation : Vertical Correct Marks : 2 Wrong Marks : 0.66 If \( f(z) = u + iv \) is an analytic function in a domain \( D \), then choose the condition that holds true for \( f(z) \) to be constant in \( D \). Options:
1. ✓ \( f'(z) \) vanishes identically in \( D \)
2. ✗ \( f'(z) \) does not vanish identically in \( D \)
3. ✗ \( \arg f(z) \) is not a constant
4. ✗ \( |f(z)| \) is not a constant
Question Number : 120  Question Id : 2310982520  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Find the inverse of a point 'a' with respect to the circle |z-c|=R.  
Options :  
1. $c + \frac{R^2}{\bar{a} - \bar{c}}$  
2. $\frac{R^2}{\bar{a} - \bar{c}}$  
3. $c - \frac{R^2}{\bar{a}}$  
4. $c + \frac{R^2}{\bar{a} + \bar{c}}$ 

Question Number : 121  Question Id : 2310982521  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Which of the following correctly describes a countable set?  
Options :  
1. A subset of a countable set is countable.  
2. A subset of a countable set is not countable.  
3. An infinite subset of a countable set is non-countable.  
4. A subset of a countable set is countable in some interval and non-countable in some interval. 

Question Number : 122  Question Id : 2310982522  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  
Correct Marks : 2  Wrong Marks : 0.66  
Find the supremum and infimum of the set $S = \{1 + (-1)^n / n : n \in \mathbb{N}\}$.  
Options :
1. √ Sup S=3/2 and inf S=0

2. ✗ Sup S=0 and inf S=3/2

3. ✗ Sup S=3/2 and inf S=1

4. ✗ No supremum and infimum exist for the given set.

Question Number : 123  Question Id : 2310982523  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Find the supremum and infimum for the set S= \((-1)^n, n \in \mathbb{N}\).

Options:
1. √ No supremum and infimum exist for the given set.

2. ✗ Sup S=-1 and inf S=6

3. ✗ Sup S=6 and inf S=-1

4. ✗ Sup S=2 and inf S=-1

Question Number : 124  Question Id : 2310982524  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
The sequence \(3^n/n!\) is:

Options:
1. ✗ monotone

2. ✗ monotone and bounded
3. ✔ bounded and convergent

4. ✗ monotone, bounded and convergent

The function $f(x)=x^2$ is uniformly continuous in:

Options:
1. ✗ $[-1, 2]$
2. ✔ $[0, 2]$
3. ✗ $[2, 2]$
4. ✔ $[-2, 2]$

Calculate the value of $c$ for the function $f(x)=(x+1)(x-2)(x+3)$ for all $x$ belonging to $[0, 1]$. Options:
1. ✔ $\frac{-2 + \sqrt{13}}{3}$
2. ✗ $\frac{-2 - \sqrt{13}}{3}$
3. ✗ $-8$
4. ✗ $-6$
Find the value of \( \int_0^1 \frac{dx}{\sqrt{1-x^2}} \).

Options:

1. □ 0

2. □ 1

3. ✔ 2

4. □ -1

Which statement of the below is true for the convergence of \( \int_0^{2a} \frac{dx}{(x-a)^2} \)?

Options:

1. ✔ The given integral does not exist and is divergent

2. □ The given integral converges to \( \pi \)

3. □ The given integral converges to \( \pi/2 \)

4. □ The given integral is oscillatory

Which of the statements below is true for the convergence of \( \int_0^\infty \cos x \, dx \)?

Options:
1. ✗ The given integral is divergent

2. ✗ The given integral converges to $\pi$

3. ✗ The given integral converges to $\pi/2$

4. ✓ The given integral is oscillatory

Calculate the value of $\lim_{n \to \infty} \frac{3+2\sqrt{n}}{\sqrt{n}}$

Options:
1. ✗ 0

2. ✗ 1

3. ✓ 2

4. ✓ 3

Find the value of $\lim_{n \to \infty} \left[ \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+n}} \right]$.

Options:
1. ✗ 0

2. ✓ 1
3. ✗ -1

4. ✗ 2

Question Number : 132  Question Id : 2310982532  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Find the value of \( \lim_{n \to \infty} \frac{1}{n} \left[ 1 + 2^{1/2} + 3^{1/3} + \cdots + n^{1/n} \right] \).

Options :
1. ✗ 0
2. ✓ 1
3. ✗ -1
4. ✗ 2

Question Number : 133  Question Id : 2310982533  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66

Which of the statements is correct for convergence of series, \( 1 - \frac{1}{3.2^2} + \frac{1}{5.3^2} - \frac{1}{7.4^2} + \cdots \) according to Abel’s test?

Options :
1. ✓ The series is Convergent
2. ✗ The series is Divergent
3. ✗ The series is Oscillating
4. ✗ The test for convergence fails

Question Number : 134  Question Id : 2310982534  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Find the continuity of \( f(x) = \sin x^2 \).

Options:

1. ✗ Uniformly continuous at \([0, \infty)\)

2. ✔ Not-uniformly continuous at \([0, \infty)\)

3. ✗ Continuous and bounded at \([0, \infty)\)

4. ✗ Second kind discontinuity at \(x=0\)

Find the value of \( \frac{\sin \alpha - \sin \beta}{\cos \beta - \cos \alpha} \) if \(0 < \alpha < \theta < \beta < \pi/2\).

Options:

1. ✗ \(\tan \theta\)

2. ✔ \(\cot \theta\)

3. ✗ \(\sin \theta\)

4. ✗ \(\cos \theta\)

Find the extreme values of the function \((x-3)^5(x+1)^4\).

Options:

1. ✔ \(-1, 3, 7/9\)
2. $-1, 3, 7$

3. $-1, 3, 9$

4. $-1, 2/5, 7/9$

Question Number : 137  Question Id : 2310982537  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : $2/3$
Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Compute the value of $\int_{1}^{2} f(x) \, dx$ using Riemann integral, where $f(x) = 3x + 1$.
Options :

1. $0$

2. $1/3$

3. $5/2$

4. $11/2$

Question Number : 138  Question Id : 2310982538  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : $2/3$
Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Compute the value of $\int_{-1}^{1} f(x) \, dx$ using Riemann integral where $f(x) = |x|$.
Options :

1. $0$

2. $-1$

3. $1$
Question Number : 139  Question Id : 2310982539  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66

Compute the value of \( \int_0^t \sin x \, dx \) using Riemann integral.

Options :

1. ✓ 1 - \cos t

2. ✗ \sin t

3. ✗ \cos t

4. ✗ \sin t - 1

Question Number : 140  Question Id : 2310982540  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66

Which of the following statements is/are true for any metric space \((X, d)\):

1. The union of an arbitrary family of open sets is open
2. The intersection of a finite number of open sets is open
3. The union of an arbitrary family of open sets is closed
4. The intersection of a finite number of open sets is closed

Options :

1. ✓ Only 1 and 2

2. ✗ Only 2 and 3

3. ✗ Only 1 and 4

4. ✗ 1, 2, 3 and 4
Which of the following statements is/are true for any metric space \((X, d)\):

1. The union of a finite number of closed sets is closed
2. The intersection of an arbitrary family of closed sets is closed
3. The union of an arbitrary family of closed sets is closed
4. The intersection of a finite number of closed sets is closed

Options:

1. ✗ 1, 2, 3 and 4

2. ✗ Only 2 and 3

3. ✗ Only 1 and 4

4. ✓ Only 1 and 2

Which of the following statement(s) hold(s) true for two subsets \(A\) and \(B\) of a metric space \((X, d)\):

1. \(\text{int } A\) is the largest open set contained in \(A\)
2. \(A\) is open if and only if \(A = \text{int } A\)
3. \(\text{Int } (A \cap B) = (\text{int } A) \cap (\text{int } B)\)

Options:

1. ✗ Only 1

2. ✗ Only 2

3. ✗ Only 2 and 3

4. ✓ 1, 2 and 3
If \( A \) is a subset of a metric space, then which of the following holds true?

Options:

1. \( \overline{A} = A \cup D(A) \)

2. \( \overline{A} = A \cap D(A) \)

3. \( \overline{A} = A \cup D(A) \)

4. \( \overline{A} = A \cup D(\overline{A}) \)

Question Number : 144  Question Id : 2310982544  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66

In a metric space, the intersection of finitely many open sets is:

Options:

1. closed

2. open

3. Open for an arbitrary family

4. Closed for a finite number

Question Number : 145  Question Id : 2310982545  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical  Correct Marks : 2  Wrong Marks : 0.66

Which statement(s) of the following is true for the convergence of Gamma function \( \int_0^{\infty} x^{n-1} e^{-x} \, dx \) ?

Options:

1. It is convergent for \( n > 0 \)

2. It is divergent for \( n \leq 0 \)
3. ✗ It is oscillatory

4. ✔ It is convergent for $n > 0$ and divergent for $n \leq 0$

Question Number : 146  Question Id : 2310982546  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Let $(a, b)$ be an open interval and $p$ be any point of $(a, b)$. The condition that every open interval is a neighbourhood of each of its points is:
Options :
1. ✔ $(p - \varepsilon, p + \varepsilon) \subseteq (a, b)$

2. ✗ $(p + \varepsilon) \subseteq (a, b)$

3. ✗ $(p - \varepsilon) \subseteq (a, b)$

4. ✗ $(p - \varepsilon, p + \varepsilon) \equiv (a, b)$

Question Number : 147  Question Id : 2310982547  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Which of the following statements is/are true for the convergence of a sequence $(X_n)$, which is monotonic increasing and bounded above?
1. It converges to its least upper bound
2. It converges to its greatest lower bound
3. It diverges to its least upper bound
4. It oscillates
Options :
1. ✔ Only 1

2. ✗ Only 2

3. ✗ Only 2 and 3
4. 2, 3 and 4

Question Number : 148  Question Id : 2310982548  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Compute the convergence of sequence $a_n = \left(1 + \frac{1}{n}\right)^n$ and the range of its limit.
Options :
1. Monotonic decreasing and bounded above, range lies between 2 and 3.
2. Monotonic decreasing and lower bounded, range lies between 1 and 3.
3. Monotonic increasing and bounded above, range lies between 2 and 3.
4. Monotonic increasing and lower bounded, range lies between 1 and 3.

Question Number : 149  Question Id : 2310982549  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
If a function is differentiable in $[a, b]$ and $f'(a)$, $f'(b)$ have opposite signs, then there exists at least one point $c \in (a, b)$ such that:
Options :
1. $f'(c)=1$
2. $f'(c)=0$
3. $f'(c)=\text{infinite}$
4. $f'(c)=f(a)$

Question Number : 150  Question Id : 2310982550  Question Type : MCQ  Option Shuffling : Yes  Negative Marks Display Text : 2/3  Option Orientation : Vertical
Correct Marks : 2  Wrong Marks : 0.66
Which statement(s) of the following is/are true about the continuity of the function \( f(x) = \tan^{-1}(1/x) \):

1. First kind discontinuity at \( x=0 \)
2. Second kind discontinuity at \( x=0 \)
3. Continuous at \( x=0 \)
4. Uniform continuous at \( x=0 \)

Options:

1. ✗ 1, 2, 3 and 4

2. ✗ Only 2, 3 and 4

3. ✔ Only 1

4. ✗ Only 1 and 2